

NEWSLETTER OF THE  
**TIMS College**  
on  
**Simulation and Gaming**



THE INSTITUTE OF  
MANAGEMENT SCIENCES

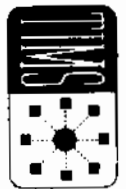
LOUIS MOORE, EDITOR and BARRY NELSON, ASSISTANT EDITOR VOL. 9 NO. 2 FALL 1985

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## CHAIRMAN'S MESSAGE

The College continues to progress toward a more organized method of operation and an increased list of activities. Arne Thesen, in conjunction with Andy Sella and a variety of others, has prepared a draft set of bylaws to replace our once-lost and outdated bylaws.

The Service Award procedure has been approved in the form published in the last Newsletter and for convenience is reprinted below. The call for nominations will appear in next spring's Newsletter.

The Outstanding Publication Award procedure has also been approved and appears in an unchanged form elsewhere in the Newsletter. This award will begin next year with the associated demise of our award for the best simulation paper in *Management Science*.

Ardayan Nozari and Allen Greenwood are investigating the possibility of the College's supporting graduate student research and, more generally, how the College can be more relevant to students.

Lee Schruben, our Vice Chairman/Chairman-Elect, has expanded our presence at the joint TMS/ORSA meeting held each spring and is maintaining our current level of activity at the fall ORSA/TMS meeting.

We continue to co-sponsor the Winter Simulation Conference.

Any ideas concerning our current activities or any proposed activities would be appreciated.

I want to thank David Kelton, who will complete his awards chairman's duties at the fall ORSA/TMS meeting when he presents the best *Management Science* Simulation Paper Award.

Bruce Schmeiser, *Chairman*

September, 1985

## MINUTES OF COLLEGE MEETING AT BOSTON TMS/ORSA CONFERENCE

Attending: Bruce Schmeiser, Barry Nelson, Ardayan Nozari, David Withers, David Kelton, Allen Greenwood, Marc Mekeon, Robert Sargent, Helen Wang, Arne Thesen, Mike Taaffe and Arnold Sweet.

College Chairman Bruce Schmeiser called the meeting to order.

1. Acting for College Secretary Jim Wilson, Barry Nelson read the minutes of the meetings that were held in Dallas in conjunction with the 1984 Fall Joint National ORSA/TMS Conference and the 1984 Winter Simulation Conference. The minutes were unanimously approved.

2. Bruce Schmeiser presented the latest "Draft Proposal for the Outstanding Service Award." After some discussion of the feasibility of giving the award every year, the proposal was adopted with 7 votes in favor and 2 abstentions.

3. Bruce Schmeiser also presented the latest "Draft Proposal for the Outstanding Simulation Publication Award." Since the purpose of the existing publication award is to encourage submission of papers to *Management Science*, there was some discussion of possible conflicts between the proposed award and this objective. The suitability of a monetary award was also discussed. David Withers moved that the proposal be accepted and that the existing award be discontinued after this year. The motion passed with 8 votes in favor and 1 abstention.

4. Arne Thesen presented the recently rediscovered College Bylaws. He and Andy Sella agreed to review the Bylaws, and they invited comments from all College members.

5. Arne Thesen also announced that *OR/MS Today* will set up a clearinghouse for software developed by members of TMS and ORSA. He noted that details are given in the August issue of *OR/MS Today*.

6. Ardayan Nozari suggested the establishment of a research award for graduate students. He and Allen Greenwood agreed to investigate this.

7. Awards Committee Chairman David Kelton encouraged all College members to submit ballots for the "Best Paper in *Management Science* Award."

Bruce Schmeiser adjourned the meeting.

Respectfully Submitted,  
James R. Wilson,  
College Secretary

The initial committee shall be appointed with assigned seniority by the College chairperson. The member in the second year shall be the chairperson. The Selection Committee shall consist of three members. Each year one new member shall be appointed by the chairperson of the TIMS/CSG for three years. The member in the second year shall be the chairperson.

5. Procedure:

- Announcement: The chairperson of the Selection Committee will call for nominations in the TIMS/CSG Newsletter and *ORMS/Today*, as well as any other medium deemed necessary, during the first six months of the calendar year.
- Nomination: Any person may nominate any person without restriction. The burden for offering evidence of merit falls on the nominator. Nominations must be in the hands of the Selection Committee chairperson by September 1.
- Selection:
  - The Selection Committee shall consist of three members. Each year one new member shall be appointed by the chairperson of the TIMS/CSG for three years. The member in the second year shall be the chairperson.

4. Funding: The plaque will be purchased from funds from the treasury of the TIMS/CSG. The only reimbursed expenses are the cost of any meals associated with the awards ceremony. There is no cash prize associated with the Award. No other expenses associated with presenting the Award will be paid.
3. Form of the Award: A plaque inscribed with the name of the Award, the year awarded, the name of recipient, and *Presented by The Institute of Management Sciences/College on Simulation and Gaming*.
2. Name of the Award: Award for Outstanding Service (hereafter called the Award).
1. Purpose of the Award: Recognize longstanding, exceptional service to the Simulation Community.

**THE OUTSTANDING SERVICE AWARD**

Respectfully Submitted,  
James R. Wilson,  
College Treasurer

As of December 31, 1984, the College had total liquid assets of \$10886.74. Thus in the first eight months of this year, the net worth of the College increased by 23%.

**ACCOUNT SUMMARY AS OF 8/31/85:**

Checking account (Purdue National)	\$ 891.17
Money market savings account (Purdue National)	4629.81
12-month 9.25% CD due 4/16/86 (Purdue National)	7585.90
Funds held at TIMS Headquarters	286.69
Total Liquid Assets	<u>\$13393.57</u>

**DISBURSEMENTS:**

Advance on the 1985 WSC	\$2000.00
College-sponsored mixer at the Boston meeting	62.16
Collection fee to close Interest checking account	5.00
Expenses for the June meeting of WSC Board of Directors	117.00
Total Disbursements	<u>\$2184.16</u>

**REVENUES:**

Interest on checking accounts (Interest Bank	\$ 152.91
NW Austin and Purdue National Bank)	119.54
Interest on money market savings (Purdue National)	400.42
Interest on 180-day 11.3% CD due 4/7/85 (Interest)	7.85
Interest on funds held at TIMS Headquarters	2000.00
Advance on the 1984 Winter Simulation Conference (WSC)	2010.27
Profit share for the 1984 WSC	<u>\$4690.99</u>
Total Revenues	

For the period from January 1 to August 31, 1985, the College had the following financial transactions:

**TREASURER'S REPORT**

- Selection Committee membership shall be offered to the last three past chairpersons of the TMS/CSG before other persons are offered membership.
- (ii) The chairperson of the Selection Committee will have full responsibility for obtaining and presenting the plaque and for arranging for publicity in the TMS/CSG Newsletter and *ORMS/Today*, as well as any other medium deemed desirable.
- (iv) The Selection Committee will make the final decision of the winner by majority vote.
- (v) The Selection Committee is to select from the set of nominees the individual who, in their opinion, has the best record of longstanding, exceptional service to the Simulation Community.
- (vi) The Selection Committee may choose to make no award.
- (vii) The Selection Committee is to make every reasonable effort to keep the names of the nominees other than that of the winners confidential.
- (viii) The Selection Committee is not to name multiple winners nor to give an honorable mention designation to any nominee.
- d. Presentation: The Selection Committee shall arrange for a suitable presentation of the Award at a national or international meeting of which TMS is a sponsor. The presentation is to be made sometime during the last three months of the year in which it is awarded, if possible.
- e. Default: The chairperson of the TMS/CSG may dissolve the Selection Committee at any time for cause. Such action may be protested by the chairperson of the Selection Committee, in which case the decision to dissolve the Selection Committee may be submitted to a vote of the TMS/CSG membership. If the Selection Committee is dissolved, the TMS/CSG chairperson shall appoint another Selection Committee. The new Selection Committee shall have the full powers of the former Selection Committee.
- ### THE OUTSTANDING SIMULATION PUBLICATION AWARD
- Purpose: To recognize outstanding contributions to the simulation literature.
- Eligibility: — Anyone is eligible to win the award.  
— Journal articles, proceedings articles, books and monographs are eligible. Technical reports, research memoranda, working papers, and dissertations/theses are not eligible. Works for which the award has been given are not eligible.
- Award: — The written work for which the award is given must be copyrighted during the three previous calendar years.  
— A plaque to each author and a \$500 prize to be divided among the authors.
- Procedure: 1. The procedures will be implemented by the TMS/CSG Awards Committee.  
One new member is appointed for a three-year term by the College chairperson each year. The member in the second year shall serve as the committee chairperson. The initial Awards Committee shall be appointed with assigned seniority by the College chairperson.  
2. Nominations will be solicited in the spring of each year via notices in the TMS/CSG Newsletter and *ORMS/Today*, as well as any other method deemed desirable by the committee.  
3. Nominations may be made by anyone, including the author(s). Nominations may not be anonymous.  
4. Nominations shall include:  
a. a copy of the written work (including all bibliographical information)  
b. a short statement suitable for reading at the award ceremony if the work is chosen  
c. and any other information thought relevant by the nominator.  
5. Nominations are closed September 1.  
6. Nominators and nominees will be held confidential by the Awards Committee, with the exception that they may consult (in confidence) with anyone they deem necessary.  
7. Only a single award may be given each year.  
8. No honorable mention awards may be made.  
9. The Awards Committee reports to the chairperson of the TMS/CSG the number of nominated papers and the winners as soon as possible after the selection and before the award ceremony.

Any person interested in simulation may associate with this College. Any member of TIMS may become a member of the College by completing a membership application and submitting it with the proper dues. All such members of the College shall have equal rights, duties and privileges.

An individual who is not a member of TIMS may become a member of the College if he/she belongs to a related society that offers reciprocal membership in its topical subdivisions to members of TIMS. Such members shall have the same rights, duties and privileges as those of regular members except for the right to hold elective office in the College.

An individual who is neither a member of TIMS nor a member of a society offering reciprocal topical subdivision membership opportunities may become a subscriber to the College. The fee for subscribers shall be equal to the dues rate for non-members and subscribers may not vote or run for office in College elections.

**TITLE III — MEMBERSHIP AND DUES**

- a. To encourage the development and dissemination of knowledge in the area of simulation.
  - b. To promote communication and interaction among individuals and organizations who share an interest in simulation.
- The College is organized and will be operated exclusively for educational and scientific purposes:

**TITLE II — PURPOSE**

This organization shall be called the College on SIMULATION.

**TITLE I — NAME**

**BYLAWS OF THE  
SPECIAL INTEREST GROUP ON  
SIMULATION  
OF THE INSTITUTE OF MANAGEMENT SCIENCES**

The proposal below will be on the agenda for discussion and first vote at the TIMS/CSG business meeting at both the ORSA/MS Atlanta Meeting and 1985 Winter Simulation Conference. The Bylaws committee consisted of Arne Theisen (chairman) and Andy Sella, and was aided by Barry Nelson, Robert Sargent, Bruce Schmeiser, Lee Schruben and David Withers.

**DRAFT PROPOSAL FOR THE BYLAWS OF THE COLLEGE ON SIMULATION**

The College's award for Best Simulation Paper appearing in *Management Science* goes this year to James R. Wilson and A. B. Pritsker for their paper "Experimental Evaluation of Variance Reduction Techniques for Queuing Simulation Using Generalized Concomitant Variables," Vol. 30, No. 12, December 1984, pp. 1459-1472. The vote was by the membership of the College, and covered five nominated papers (covering the 1983 as well as the 1984 Volume years, due to lack of three nominations in 1983). The award consists of a \$250 check and a plaque to each author, presented at the Awards Luncheon at the Fall 1985 ORSA/TIMS Conference in Atlanta.

**1983-84 BEST PAPER AWARD**

- 10. The winning author is notified as soon as possible after the selection and invited to the awards ceremony by the awards chairman.
- 11. The awards chairman contacts the program chairperson of the spring Joint National TIMS/ORSA meeting and requests a spot in the awards ceremony.
- 12. The awards chairman ensures that plaques and checks are available at the ceremony. No expenses other than meals at the awards ceremony shall be reimbursed.
- 13. The award is presented at the awards ceremony at the spring Joint National Meeting of ORSA and TIMS by the chairperson of the Awards Committee or a substitute named by the TIMS/CSG chairperson.
- 14. The winner is announced in both the TIMS/CSG Newsletter and *ORMS/Today*, as well as in any other medium deemed desirable by the Awards Committee.
- 15. The decision of the Awards Committee is final.
- 16. \*Default: The chairperson of the TIMS/CSG may dissolve the Awards Committee at any time for cause. Such action may be protested by the chairperson of the Awards Committee, in which case the decision to dissolve the Awards Committee may be submitted to a vote of the TIMS/CSG membership. If the Awards Committee is dissolved, the TIMS/CSG chairperson shall appoint another Awards Committee. The new Awards Committee shall have the full powers of the former Awards Committee.

1. The officers of the College shall consist of a President, a Vice President, a Secretary and a Treasurer. The Executive Committee shall consist of the officers, the immediate Past President and the Chairs of the standing committees.
2. The President shall be the chief administrative officer of the College, shall preside at all business meetings of the College and its Executive Committee, shall manage the affairs of the College between meetings, and shall appoint all committees and committee chairpersons of the College.
3. The Vice President shall assume the duties of the President in the event of the President's absence. The Vice President also shall plan all program meetings and arrange for all business and program meetings of the College.
4. The Secretary shall keep minutes of all College and Executive Committee business meetings, send all notices to the membership and Executive Committee, conduct College elections, and perform other duties usual to the office of Secretary. The Secretary shall maintain needed contact with and supply needed reports to TIMS.
5. The Treasurer shall work with the appropriate Business Offices to ensure that dues are collected and mailing lists are maintained, maintain financial and membership records, supervise disbursements of funds, arrange for any bank accounts needed to handle College funds with approval of the Executive Committee for the banks selected, and perform other duties usual to the office of Treasurer. The Treasurer shall maintain needed contact with and supply needed reports to TIMS.
6. Each officer must be a member of TIMS and the College.
7. The term of office shall begin June 1 of even numbered years.
8. Vacancies in any office may be filled at the discretion of the Executive Committee until the next scheduled election.

ARTICLE IV — OFFICERS AND THEIR DUTIES

ARTICLE V — ELECTIONS

1. The Nominating Committee shall select at least two nominees for each office and submit the list of nominees to the Secretary for publication in the fall Newsletter. Any group of 10 or more members may then nominate a candidate for any office by submitting a signed petition to the Secretary-Treasurer no later than February 1. The spring Newsletter will include the ballots, showing all nominees listed in alphabetical order with no distinction between petition and Nominating Committee candidates and providing space for write-in candidates. The ballots shall specify the deadline for their return, which shall be the same as the date of the spring business meeting. Ballot counting shall be the responsibility of the Nominating Committee.
- In the event of a tie vote for any position, the members of the Executive Committee shall choose between the tied candidates by written ballot. Results of the election shall be communicated in the fall Newsletter.
2. A plurality of qualified votes cast shall be necessary for election.
3. The Vice President shall succeed to the office of President in the following term. If it is known before the Nominating Committee submits its list of nominees that the Vice President will be unable to succeed to the office of President, then nominations shall be solicited for the office of President.

ARTICLE VI — MEETINGS AND PROGRAMS

1. Business meetings of the College shall be held at least twice each year in conjunction with the fall and spring meetings of TIMS and the Winter Simulation Conference. A quorum for business meetings shall be 15 members or five percent of the total membership, whichever is less.
2. The Executive Committee shall meet at least twice a year, at the call of the President. The quorum for the Executive Committee meetings shall consist of half its membership.
3. Each Newsletter shall include the minutes of the preceding business meeting(s) (if any) and the agenda for the next business meeting.
4. Roberts' Rules (Revised) shall be followed in all meetings, except as they conflict with these Bylaws.

ARTICLE VII — COMMITTEES

1. The Standing Committees and their duties shall be as follows:
  - (a) Award/Publication — handles all matters with respect to the Outstanding Simulation Publication Award.
  - (b) Award/Service — handles all matters with respect to the Outstanding Service Award.
  - (c) Publications — oversees the Newsletter and handles publicizing of activities to college members and the general public.
  - (d) Nominating — nominates candidates for office and counts ballots.
2. The President may create ad hoc committees and appoint their members and chairpersons. The tenure of an ad hoc committee shall expire with the term of the President who appointed it.

ARTICLE VIII — AMENDMENTS TO THE BYLAWS

1. Amendments to these Bylaws must be approved by:

(a) the College membership,

(b) the appropriate oversight committee within TMS.

An amendment shall be effective when and only when both of these required approvals have been obtained.

2. Approval of an amendment by the membership must be obtained in the following manner:

(a) A proposed amendment may be submitted for action by vote of the Executive Committee or by written petition of 15 members or five percent of the total membership, whichever is less.

(b) The amendment shall be read and discussed at a College business meeting. Notice of the meeting and the wording of the proposed amendment shall be distributed to the members at least 30 days before the meeting.

(c) With the first Newsletter following the meeting at which the amendment was discussed, the Secretary shall distribute mail ballots on the amendment. These ballots shall include the date by which ballots must be returned, which shall be at least six weeks after the date the ballots are distributed. A two-thirds majority constituting at least 10% of all qualified members shall be required for approval.

NEW BOOKS

Elements of Simulation, B. J. T. Morgan, 1984, Chapman and Hall, NY.

Available from Methuen, Inc., 733 Third Avenue, New York, NY 10017, \$45.00 hardbound, \$18.95 paperback.

Table of Contents: Introduction, Some probability and statistics revision, Generating uniform random variables, Particular methods for non-uniform random variables, General methods for non-uniform random variables, Testing random numbers, Variance reduction and integral estimation, Model construction and analysis, Further examples and applications.

ABSTRACTS OF PAPERS

COMPUTER GENERATION OF HYPERGEOMETRIC RANDOM VARIABLES, to appear in *Journal of Statistical Computation and Simulation*, Vioras Kachitvichyanukul, Department in Industrial and Management Engineering, The University of Iowa, Iowa City, Iowa 52242; and Bruce W. Schneiser, School of Industrial Engineering, Purdue University, West Lafayette, Indiana 47907.

Abstract: The paper presents an exact, uniformly fast algorithm for generating random variables from the hypergeometric distribution. The overall algorithm framework is acceptance/rejection and is implemented via composition. Three subdensities are used, one is uniform and the other two are exponential. The algorithm is compared with algorithms based on sampling without replacement, inversion, and aliasing. A comprehensive survey of existing algorithms is also given.

ON CONTROL VARIATE ESTIMATORS, Working Paper Series No. 1985-006, Barry L. Nelson, Department of Industrial and Systems Engineering, The Ohio State University, Columbus, Ohio 43210.

Abstract: Control Variate (CV) estimators are a useful class of reduced variance estimators for simulation experiments because they correct the value of a crude estimator after the simulation outputs have been realized, and thus they do not alter the time path of the simulated stochastic process. In this note, a broad definition of CV estimators is proposed, the properties of this class of estimators are investigated, and the definition is employed to derive a new CV estimator that is useful for estimating probabilities.

DISCRETE-TIME CONVERSION FOR SIMULATING SEMI-MARKOV PROCESSES, Bennett L. Fox, Department d'Informatique et de recherche operationnelle, Universite de Montreal; and Peter W. Glynn, Department of Industrial Engineering, University of Wisconsin.

Abstract: To simulate long-run averages of time integrals of a recurrent semi-Markov process efficiently, convert to discrete-time by simulating only an imbedded chain and computing the conditional expectations of everything else needed, given the sequence of states visited. This reduces asymptotic variance and eliminates generation holding-time variates. In this setting, uniformizing continuous-time Markov chains is not worthwhile. We generalize beyond semi-Markov processes and cut ties to regenerative simulation methodology.

CONFIDENCE INTERVALS BASED ON STEADY-STATE CONTINUOUS-TIME STATISTICS, A. Nozari, AT & T Bell Laboratories, Murray Hill, New Jersey 07974.

Abstract: In steady-state computer simulation experiments, continuous-time statistics are used to estimate a variety of systems parameters. In this paper we present formulas for estimating confidence intervals based on such statistics. Our formulas are extensions of the work of Schruben on "Standardized Time Series" for discrete-time observations. We conduct a Monte Carlo study to examine the behavior of our estimators. Our estimators work quite well and our results are consistent with those of Schruben for discrete-time observations.

CONFERENCE CALENDAR

Upcoming Conference Sessions Sponsored by the TIMS/CSCG:  
 ORSA/TIMS Atlanta, November 4-6, 1985, Atlanta Hilton and Towers.  
 MB18 "Statistical Methodology for Simulation," Andrew F. Scilla, University of Georgia, chairperson.  
 MC18 "Statistical Aspects of Simulation," Robert G. Sargent, Syracuse University, chairperson.  
 MD18 "Simulation Methodology," David Kelton, University of Michigan, chairperson.  
 Also of interest to TIMS/CSCG members attending the Atlanta Conference:  
 TA18 "Simulation I," Jerry Banks, Georgia Institute of Technology, chairperson.  
 TB18 "Simulation II: Cases," John S. Carson, Georgia Institute of Technology, chairperson.  
 TC18 "Simulation III," James J. Swain, Georgia Institute of Technology, chairperson.  
 The TIMS/CSCG will again be sponsoring a mixer at the Atlanta Conference. The mixer will be Tuesday, November 5th at 9 p.m., at a location to be announced, and is open to everyone.  
 TMSA/ORSA Los Angeles, April 14-16, 1986, The Bonaventure Hotel.  
 "Data Structures for Simulation," David Withers, IBM, chairperson.  
 "Tutorial on Manufacturing Simulation," Averill M. Law, University of Arizona, chairperson.  
 "Variance Reduction in Simulation," Bruce Schmeiser, Naval Postgraduate School, chairperson.  
 "Design of Simulation Experiments," Ardavan Nozari, AT&T Bell Laboratories, chairperson.  
 "Standardized Time Series," David Goldman, Georgia Institute of Technology, chairperson.  
 "Simulation of Non-Markovian Systems," Donald L. Iglehart, Stanford University, chairperson.  
 "Animation of Simulated Systems," C. Dennis Pegden, Systems Modeling Corporation, chairperson.  
 "Solving Industrial Problems with Simulation," Charles R. Standridge, Pritsker and Associates, chairperson.  
 The TIMS/CSCG is one of the sponsors of the 1985 Winter Simulation Conference, San Francisco Hilton and Towers, December 11-13, Gerard C. Blais, General Chairman (202) 274-8015.  
 Also of interest to TIMS/CSCG members:  
 CORSA/SCRO in Toronto, June 2-4, 1986, will have a session entitled "Simulation and Modeling," Bennett L. Fox, Université de Montréal, chairperson.  
 Speakers: Luc Devroye, Peter Glynn and Lee Schruben.  
 1986 SCS Multiconference, January 23-25, 1986, San Diego, CA.  
 Contact Barbara Atkinson, SCS, P. O. Box 17900, San Diego, CA 92117.  
 Eastern Simulation Conference, March 10-12, 1986, Norfolk, VA.  
 Contact Barbara Atkinson (see above).  
 Nineteenth Annual Simulation Symposium, March 12-14, 1986, Tampa, FL.  
 Contact Annual Simulation Symposium, P. O. Box 22621, Tampa, FL 33622.

EDITOR'S CORNER

THE SAMPLE VARIANCE OF ANTIHETIC VARIATES, Barry L. Nelson, Department of Industrial and Systems Engineering, The Ohio State University, Columbus, Ohio.

Let  $\{(X_i, Y_i), i = 1, 2, \dots, n\}$  be antihetic pairs of random variables, by which we mean

$$E\{X_i\} = E\{Y_i\} = \theta$$

$$\text{Cov}\{X_i, X_j\} = \text{Cov}\{Y_i, Y_j\} = \sigma^2 \text{ if } i = j, \text{ and } 0 \text{ if } i \neq j$$

$$\text{Cov}\{X_i, Y_j\} = \rho\sigma^2 \text{ if } i = j, \text{ and } 0 \text{ if } i \neq j$$

where  $\rho < 0$ . Usually we average antihetic pairs, letting  $Z_i = (X_i + Y_i)/2$ , so that  $Z = n^{-1}\sum Z_i$  is an estimator of  $\theta$ . An unbiased estimator of the variance of  $Z$  is

$$[n(n-1)]^{-1} \sum (Z_i - Z)^2$$

Now it is well known that

$$\text{Var}\{Z\} = 1/4\{\text{Var}\{X\} + \text{Var}\{Y\} + 2\text{Cov}\{X, Y\}\}$$

where  $\bar{X} = n^{-1}\sum X_i$  and  $\bar{Y} = n^{-1}\sum Y_i$ . This suggests the direct estimator of  $\text{Var}\{Z\}$  given by (2):

$$(2) \quad [4n(n-1)]^{-1} \sum (X_i - \bar{X})^2 + \sum (Y_i - \bar{Y})^2 + [2n(n-1)]^{-1} \sum (X_i Y_i - n\bar{X}\bar{Y})$$

Students are sometimes more receptive to (2) initially because it seems to directly account for the induced correlation by including an estimate of the covariance. But are (1) and (2) different?

Result: (1) and (2) are algebraically equivalent. The proof is simply to expand (1), substitute for  $Z_i$  and (tediously) collect terms.

Send all contributions to the TIMS/CSCG Newsletter to either Louis Moore, The Rand Corporation, 1700 Main Street, Santa Monica, CA 90406, or Barry L. Nelson, Department of Industrial and Systems Engineering, The Ohio State University, Columbus, OH 43210.